

Discussion 03

Weekly Material Recap and Example Problems

Instructor: Tim Carlson**Contact:** timothy.carlson@berkeley.edu**Date:** September 17, 2026**Version:** Instructor copy

1 General reminders

- **Assignment reminder:** Weekly homework is due on Fridays at 6:00 pm; extensions are possible under extenuating circumstances, but please notify me/the GSI team if such an extension is required.
- **Free body diagrams:** Many students are not drawing free body diagrams. This can lead to confusion and error, so I recommend that you do this as a part of the solution process. It is important to know how to do this in general, even if you do not do it for every problem you solve.
- **Midterm 1:** The first midterm will be this coming Monday during class time, covering all material through friction.
- **GSI logistics reminder:** My office hours are 3:30-5:00 pm Mondays and Thursdays in 347 Soda. Discussion section is 5:00-6:00 pm in 306 Soda.

2 September 7–September 11: material recap

2.1 Equivalent force systems

We have discussed equivalent force systems before: two systems are **equivalent** if they have the same resultant force and the same moment about a point. They therefore have the same effect on the equilibrium of a rigid body.

Equivalent force systems

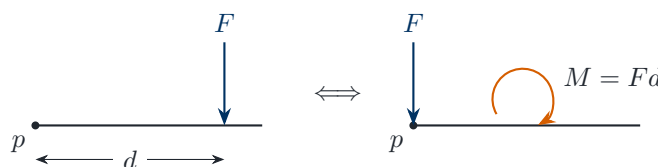
Consider forces $\{\mathbf{F}_i\}$ and $\{\mathbf{G}_j\}$. The systems are equivalent if

$$\sum_i \mathbf{F}_i = \sum_j \mathbf{G}_j, \quad \sum_i \mathbf{r}_{pi} \times \mathbf{F}_i = \sum_j \mathbf{r}_{pj} \times \mathbf{G}_j,$$

where $\mathbf{r}_{pi}$ points from p to the application point of $\mathbf{F}_i$. Include any applied couple moments in the moment sums.

If both equalities hold at **one point**, they hold at every point.

A force system can always be replaced by its resultant force applied at p and a couple equal to its moment about p . Sometimes a single force is sufficient, but its line of action must also reproduce the original moment.



Moving the force to p requires a clockwise couple of magnitude Fd .

2.2 Distributed loads

Nearly all engineering loads are distributed, although it is often useful to approximate them as point loads. A distributed load is simply force spread over a line, area, or volume.

Resultant of a distributed load

For a force per unit area $\mathbf{f}$ [N/m²] acting over $\mathcal{B}$, the resultant force and moment about p are

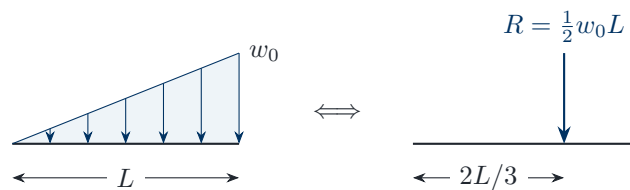
$$\mathbf{R} = \int_{\mathcal{B}} \mathbf{f} dA, \quad \mathbf{M}_p = \int_{\mathcal{B}} (\mathbf{x} - \mathbf{x}_p) \times \mathbf{f} dA.$$

The integrals are computed component-wise. For line or volume loads, replace dA by dL or dV and use force densities in N/m or N/m³, respectively.

To locate the equivalent point force, we match both the total force and the moment of the distributed load. For the downward beam load below, the moment magnitudes about the origin must satisfy

$$Rx_c = \int_0^L xw(x) dx.$$

Dividing by R gives the expression for x_c .



The triangular load acts at $2L/3$ from its zero-load end.

The same idea applies to a load distributed over a flat area. Suppose the area lies in the xy -plane and the load acts normal to it, with $\mathbf{f} = w(x, y)\mathbf{e}_z$. The resultant is $\mathbf{R} = R_z\mathbf{e}_z$, where

$$R_z = \int_A w(x, y) dA.$$

If $R_z \neq 0$, applying this force at the equivalent loading point $(x_c, y_c, 0)$ must reproduce the distributed load's moments about both the x - and y -axes:

$$y_c R_z = \int_A yw(x, y) dA, \quad -x_c R_z = - \int_A xw(x, y) dA.$$

In both the 2-D and 3-D cases, if the resultant force is zero but the moment is nonzero, then an equivalent force cannot be found; rather, the equivalent system contains 2 equal and opposite forces or a single couple moment.

Complicated load diagrams can often be split into simple shapes. It is often easier to compute each part's resultant, then add their forces and moments using the **same reference point** to obtain the equivalent loading.

2.3 Centroids of composite shapes

The centroid is the geometric balance point of a shape. For a thin plate of uniform thickness and density, it is also the center of mass.

Area centroid

For an area A , the centroid is

$$\mathbf{x}_c = \frac{1}{A} \int_A \mathbf{x} dA.$$

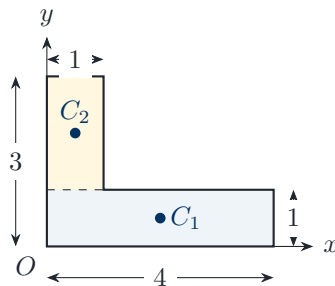
If the shape is divided into simple parts,

$$x_c = \frac{\sum_k A_k x_k}{\sum_k A_k}, \quad y_c = \frac{\sum_k A_k y_k}{\sum_k A_k},$$

where A_k is the area of part k and (x_k, y_k) is its centroid, measured from a **common origin**.

Use nonoverlapping parts so that no area is counted twice. Holes can be treated as negative areas. A rectangle's centroid is at its center; a triangle's centroid is one-third of the altitude from its base.

For example, split the following L-shaped area into two rectangles. All lengths are in meters.



Part	A_k	x_k	y_k
Bottom rectangle	4	2	1/2
Upper rectangle	2	1/2	2

$$x_c = \frac{4(2) + 2(1/2)}{6} = 1.5 \text{ m},$$

$$y_c = \frac{4(1/2) + 2(2)}{6} = 1 \text{ m}.$$

2.4 Multibody equilibrium

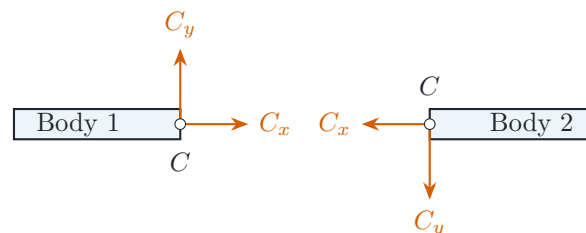
For connected bodies, we can apply equilibrium to the **whole assembly** or to **each body separately**. The whole assembly is useful for finding external reactions; separating the bodies reveals member forces.

Equilibrium of connected bodies

For each body in planar equilibrium,

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_p = 0.$$

Forces at a connection are **equal and opposite** on the two bodies. They cancel in the free body diagram of the whole assembly, but must appear when the bodies are isolated.



Free body diagrams separated at the internal pin C ; only the pin forces are shown.

Draw a free body diagram before writing each body's equations. A very common case you will encounter is that of a truss: in an ideal truss, straight members are connected by pins, loads act only at the joints, and member weights are neglected. Each member is therefore a two-force member and carries only axial tension or compression.

3 Example problems

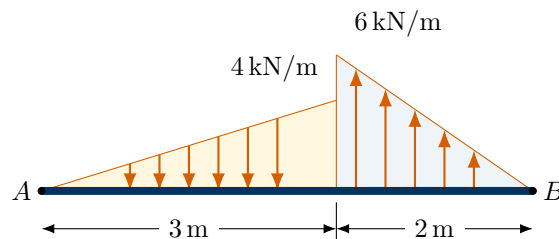
Report answers with appropriate units. Draw a free body diagram where needed.

Example 1: A distributed load that changes direction

A distributed load acts along a straight beam AB of length 5 m. Over the first 3 m, the load acts downward and increases linearly from zero at A to 4 kN/m. Over the remaining 2 m, it acts upward and decreases linearly from 6 kN/m to zero at B .

Take x positive to the right, measured from A , and y positive upward. The arrows indicate the load direction.

1. Calculate the resultant force and moment about A . Can this distribution be replaced by a single point force? Explain.
2. Find the minimum number of point forces needed to represent the distribution, without using an applied couple. Give their magnitudes, directions, and one valid choice of application points on the beam.



Solution

Each triangular load is replaced by a force equal to its area, acting through its centroid.
For the downward load,

$$R_1 = \frac{1}{2}(3\text{ m})(4\text{ kN/m}) = 6\text{ kN}, \quad x_1 = \frac{2}{3}(3\text{ m}) = 2\text{ m}.$$

For the upward load, the centroid is one-third of its length from the end of greatest intensity:

$$R_2 = \frac{1}{2}(2\text{ m})(6\text{ kN/m}) = 6\text{ kN}, \quad x_2 = 3\text{ m} + \frac{1}{3}(2\text{ m}) = \frac{11}{3}\text{ m}.$$

Thus, the resultant force is

$$\mathbf{R} = (-6 + 6)\mathbf{e}_y\text{ kN} = \mathbf{0}.$$

Taking counterclockwise moments as positive,

$$M_{A,z} = -(6\text{ kN})(2\text{ m}) + (6\text{ kN})\left(\frac{11}{3}\text{ m}\right) = 10\text{ kN m}.$$

A single point force cannot reproduce this system: it would have to equal the zero resultant, but a zero force cannot produce the nonzero moment. The distribution is equivalent to a pure couple.

The minimum is therefore **two point forces**. One valid choice is

$$\mathbf{F}_1 = -6\mathbf{e}_y\text{ kN} \quad \text{at } x_1 = 2\text{ m}; \quad \mathbf{F}_2 = 6\mathbf{e}_y\text{ kN} \quad \text{at } x_2 = \frac{11}{3}\text{ m}.$$

The choice is not unique. Any downward force on the left and equal upward force on the right with

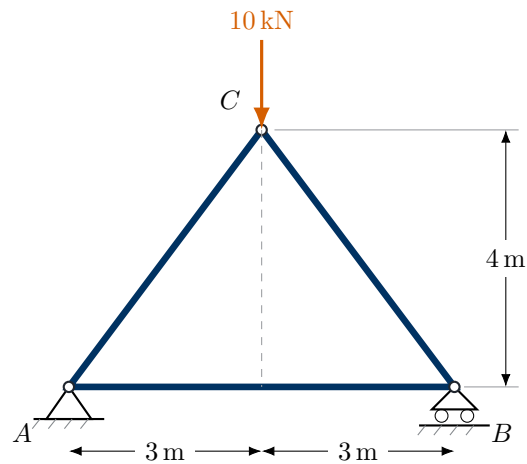
$$F(x_2 - x_1) = 10\text{ kN m}$$

produces the same resultant force and moment.

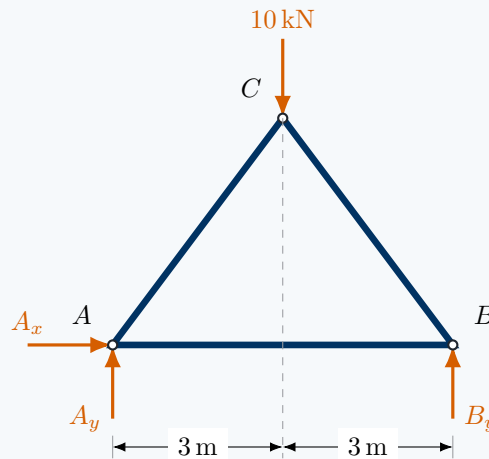
Example 2: A three-member truss

The truss below consists of three straight, weightless members connected by ideal pins at A , B , and C . It is supported by a pin at A and a roller on a horizontal surface at B . A downward force of 10 kN acts at C .

1. Determine the support reactions.
2. Determine the force in each member and state whether it is in tension or compression.



Solution



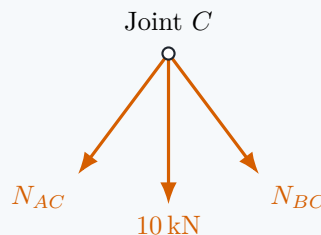
The whole truss is represented by the free body diagram above. Taking moments about A gives

$$B_y(6\text{ m}) - (10\text{ kN})(3\text{ m}) = 0 \implies B_y = 5\text{ kN}.$$

Force equilibrium then gives

$$A_x = 0, \quad A_y = B_y = 5\text{ kN upward}.$$

Each member is a truss (two-force) member, so its force acts along the line joining its pins. At joint C , assume both member forces act away from the joint (tension).



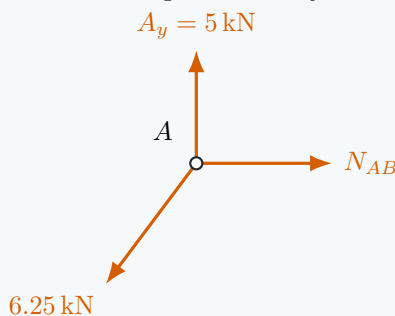
Horizontal and vertical equilibrium give

$$-\frac{3}{5}N_{AC} + \frac{3}{5}N_{BC} = 0, \quad \text{and} \quad -\frac{4}{5}N_{AC} - \frac{4}{5}N_{BC} - 10\text{ kN} = 0,$$

respectively. $N_{AC} = N_{BC}$ is negative, indicating that our tension assumption was wrong; therefore,

$$AC, BC : 6.25\text{ kN in compression}.$$

At joint A , the nonzero joint forces can be represented by



Horizontal equilibrium gives

$$N_{AB} - \frac{3}{5}(6.25\text{ kN}) = 0, \implies AB : 3.75\text{ kN in tension}.$$

4 Challenge exercise

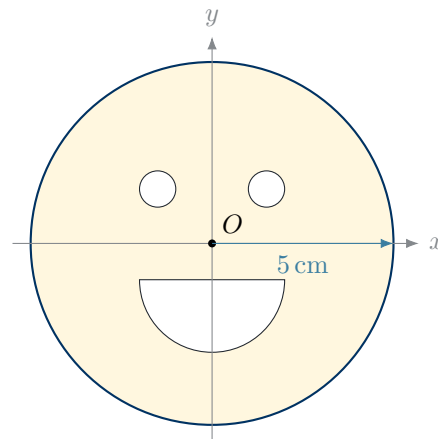
Example 3: The centroid of a smiley face

A thin plate of uniform thickness and density is made from a circular disk of radius 5 cm by removing two circular eyes and a semicircular mouth.

The origin is at the center of the original disk. The cutouts are:

- Two circular eyes, each of radius 0.5 cm, centered at $(-1.5, 1.5)$ cm and $(1.5, 1.5)$ cm.
- A semicircular mouth of radius 2 cm, with its horizontal diameter centered at $(0, -1)$ cm. The mouth lies **below** its diameter.

Find the centroid (x_c, y_c) of the remaining plate.



White regions are removed from the plate.

Solution

Symmetry about the y -axis gives

$$\boxed{x_c = 0}.$$

For the eyes and circular plate, the centroid is clear given that the intersection of all lines of symmetry in a circle is the center. However, we must still compute the centroid of the half circle. This is most easily done in polar coordinates, where $y = r \sin \theta$ (relative to the circle center) and $dy dx = r dr d\theta$. Thus,

$$y_k = -1 - \frac{\int_0^\pi \int_0^2 r \sin \theta (r dr d\theta)}{\frac{1}{2}\pi(2)^2} = -1 - \frac{4}{3\pi} \int_0^\pi \sin \theta d\theta = -1 - \frac{8}{3\pi} \quad [\text{cm}].$$

Per the remark in the handout, we treat the removed area as negative:

Part	A_k [cm ²]	y_k [cm]
Full disk	25π	0
Both eyes (combined)	$-\pi/2$	$3/2$
Mouth	-2π	$-1 - 8/(3\pi)$

Thus,

$$y_c = \frac{(25\pi)(0) - (\pi/2)(3/2) + (2\pi)(1 + 8/(3\pi))}{25\pi - \pi/2 - 2\pi} \text{ cm.}$$

Simplifying,

$$\boxed{y_c = \frac{5\pi + 64/3}{90\pi} \text{ cm} = \left(\frac{1}{18} + \frac{32}{135\pi} \right) \text{ cm} \approx 0.131 \text{ cm.}}$$