CE 30 / ME 85 — Introduction to Solid Mechanics **Fall 2026**

Discussion 06

Weekly Material Recap and Example Problems

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Adapted from Ryan McAvoy's Discussion 6 handout. Thanks for covering!

1 General reminders

- **Material coverage:** Last week we covered deformation of one-dimensional bars. This week we will cover stress.
- **Additional resource:** Ryan recommends `https://www.continuummechanics.org` as a helpful resource for learning about stress.
- **Reading coverage:** Lecture has covered Chapters 1–6 and 9 of [H], Sections 1–3.3 of [G], and the lecture handout.

2 September 28–October 2: material recap

2.1 Stress

Stress is a measure of force per unit area. In a one-dimensional context, stress is a scalar:

$$\sigma = \frac{F}{A}.$$

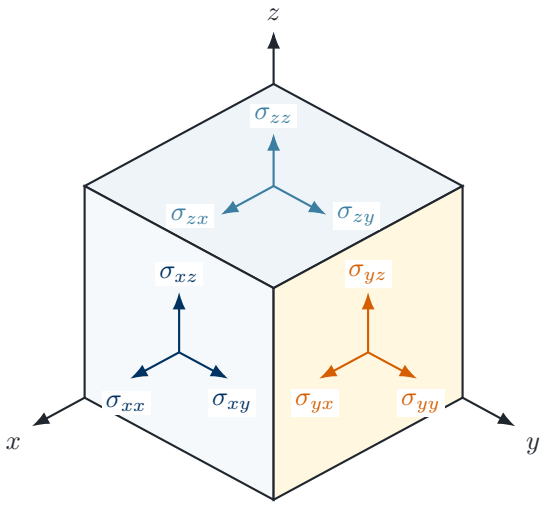
In three dimensions, stress is a second-order tensor with nine scalar components.

- A second-order tensor maps a vector to a vector.
- Just as a three-dimensional vector has three components in a chosen basis, a second-order tensor has $3 \times 3 = 9$ components.
- These components can be represented by a matrix once a basis is chosen.

We write the stress components as

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}.$$

Here, σ_{ij} denotes the stress component in direction $\boldsymbol{e}_j$ on a face with unit normal $\boldsymbol{e}_i$. Thus, the rows correspond to the x-, y-, and z-faces.



Stress components on the positive coordinate faces.

2.2 Traction, normal stress, and shear stress

Traction on a plane

For a plane with unit normal $\boldsymbol{n}$, the traction (force per unit area) is

$$\boldsymbol{t} = \boldsymbol{\sigma}^T \boldsymbol{n},$$

or, in components,

$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}^T \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}.$$

The normal stress N and shear stress magnitude τ are

$$\boxed{N = \boldsymbol{t} \cdot \boldsymbol{n}, \qquad \tau = |\boldsymbol{t} - N\boldsymbol{n}|\,.}$$

2.3 Stress equilibrium

Force balance, including a body force per unit volume with components b_x, b_y, b_z, gives

Equilibrium equations

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + b_x = 0, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + b_y = 0, \\ \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0. \end{aligned}$$

Moment balance requires the stress tensor to be symmetric, as we will assume throughout this course:

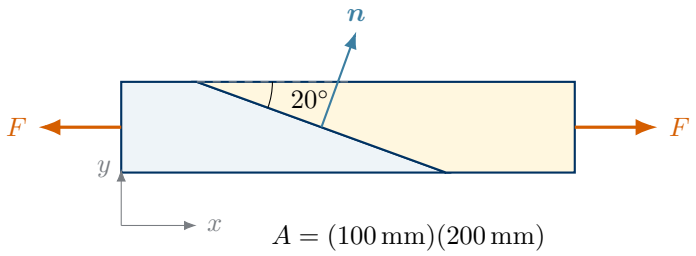
$$\sigma_{xy} = \sigma_{yx}, \qquad \sigma_{xz} = \sigma_{zx}, \qquad \sigma_{yz} = \sigma_{zy}.$$

3 Example problems

Report answers with appropriate units. Draw a free body diagram where applicable.

Example 1: An angled glued joint ([G], 3.1)

Two pieces of wood are glued together along a joint inclined at 20° to the horizontal. The member has a rectangular cross section measuring 100 mm × 200 mm.
The shear strength of the glue is 10 N/mm^2. Find the maximum permissible tensile load F based on shear failure of the glue.



$A = (100\,\text{mm})(200\,\text{mm})$

Side view of the joint.

Solution

The member is in uniaxial tension, so its in-plane stress components are

$$[\sigma_{ij}] = \begin{bmatrix} F/A & 0 \\ 0 & 0 \end{bmatrix}, \qquad A = 20{,}000\,\text{mm}^2.$$

The joint has unit normal

$$\boldsymbol{n} = \sin 20^\circ\, \boldsymbol{e}_x + \cos 20^\circ\, \boldsymbol{e}_y.$$

Thus,

$$\boldsymbol{t} = \boldsymbol{\sigma}^T \boldsymbol{n} = \frac{F}{A} \sin 20^\circ\, \boldsymbol{e}_x, \qquad N = \boldsymbol{t} \cdot \boldsymbol{n} = \frac{F}{A} \sin^2 20^\circ.$$

The shear stress magnitude is

$$\tau = |\boldsymbol{t} - N\boldsymbol{n}| = \frac{F}{A} \sin 20^\circ \cos 20^\circ.$$

Setting this equal to the glue's shear strength gives

$$\boxed{F_{\max} = \frac{(10\,\text{N/mm}^2)(20{,}000\,\text{mm}^2)}{\sin 20^\circ \cos 20^\circ} \approx 622\,\text{kN}.}$$

Example 2: Hydrostatic stress ([G], 3.9)

Consider the hydrostatic stress state

$$[\sigma_{ij}] = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{MPa}.$$

For an arbitrary plane through the point, determine the force per unit area acting parallel and normal to the plane.

Solution

Let $\boldsymbol{n}$ be the unit normal to the plane. Then

$$\boldsymbol{t} = \boldsymbol{\sigma}^T \boldsymbol{n} = (10\,\text{MPa})\boldsymbol{n}.$$

Since $\boldsymbol{n} \cdot \boldsymbol{n} = 1$,

$$\boxed{N = \boldsymbol{t} \cdot \boldsymbol{n} = 10\,\text{MPa}}, \qquad \boxed{\tau = |\boldsymbol{t} - N\boldsymbol{n}| = 0}.$$

Every plane has the same normal stress and zero shear stress. This is an **isotropic stress state**.

Example 3: Stress equilibrium

Determine whether the following stress fields satisfy equilibrium at $\boldsymbol{x} = (1, 1, 0)$. Assume $\boldsymbol{b} = \boldsymbol{0}$.

$$\boldsymbol{\sigma}_1 = \begin{bmatrix} 3x & 2x^3 & -3xz \\ 2x^3 & -6y & -z^2 \\ -3xz & -2x^3 & 0 \end{bmatrix}, \qquad \boldsymbol{\sigma}_2 = \begin{bmatrix} 3x & 2x^3 & -3xz \\ 2x^3 & -6y & -z^2 \\ -3xz & -z^2 & 0 \end{bmatrix}.$$

Solution

Both force and moment balance must hold.
For $\boldsymbol{\sigma}_1$, at $(1, 1, 0)$,

$$\sigma_{yz} = 0, \qquad \sigma_{zy} = -2.$$

The stress is not symmetric, so moment balance fails.
The second field $\boldsymbol{\sigma}_2$ is symmetric. Checking force balance gives

$$\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} &= 3 + 0 - 3x, \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} &= 6x^2 - 6 - 2z, \\ \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= -3z + 0 + 0. \end{aligned}$$

All three expressions vanish at $(1, 1, 0)$. Therefore,

$$\boxed{\boldsymbol{\sigma}_2 \text{ satisfies equilibrium at } (1, 1, 0), \quad \boldsymbol{\sigma}_1 \text{ does not.}}$$

References

[H] Dietmar Gross, Werner Hauger, Jörg Schröder, Wolfgang A. Wall, and Nimal Rajapakse. *Statics: Engineering Mechanics 1.* 2nd edition, Springer, 2013.

[G] Sanjay Govindjee. *Engineering Mechanics of Deformable Solids: A Presentation with Exercises.* Oxford University Press, 2013.